WBGMC Answers

The Williston Northampton School

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1 Answers

1. Assume there are 100 fruits in total for convenience sake, there would be:
   25 blueberries
   15 strawberries
   30 apples
   5 pineapples
   50 peaches

Since the question asks about the percent of apples in fruits which are not berries, the numerator is the number of apples, and the denominator is the sum of pineapples, peaches, and apples, which would be

$$\frac{30}{100-(25+15)}=\frac{1}{2}$$

2. Assume the larger number as a and the smaller number as b:
   Set up the equation

$$a+b=7\times(a-b)$$

   We get

$$-6\times a=-8\times b$$

   The ratio would be

$$\frac{a}{b}=\frac{4}{3}$$

3. There are $12\times 12=144$ numbers in total. Only two odd numbers produce an odd product. 6 of the 12 numbers are odd, which would yield 6×6 odd numbers in total. Therefore, the number of even products would be:

$$144-36=108$$

4. the x and y intercepts of the graph is (6,0) and (0,8). Connecting the two points produces a right triangle with a base of 6 and a height 8, which are two of the heights

of the triangle. The third height can be found by first finding the length of the third side of the triangle using the Pythagorean Theorem:

$$\sqrt{6^2 + 8^2} = 10$$

Then finding the area of the triangle with the known two legs:

$$\frac{6 \times 8}{2} = 24$$

The third height, denoted by $h3$, can be found with:

$$\frac{h3 \times 10}{2} = 24$$

$$h3 = 4.8$$

The sum of three heights is in turn:

$$6 + 8 + 4.8 = 18.8$$

5. Floor functions are used to solve this question because they denote the maximum number of multiples equal to or less than 2021. First find the multiples of 3 and 5:

$$\left\lfloor \frac{2021}{3} \right\rfloor + \left\lfloor \frac{2021}{5} \right\rfloor - \left\lfloor \frac{2021}{3 \times 5} \right\rfloor$$

Since the multiples of 7 should be excluded, subtract the answer above with 3×7 and 5×7:

$$\left\lfloor \frac{2021}{3} \right\rfloor + \left\lfloor \frac{2021}{5} \right\rfloor - \left\lfloor \frac{2021}{3 \times 5} \right\rfloor - \left\lfloor \frac{2021}{3 \times 7} \right\rfloor - \left\lfloor \frac{2021}{5 \times 7} \right\rfloor$$

Lastly, the multiples of 3, 5, and 7 were subtracted twice from the previous answer, so add back the multiples of 3, 5, and 7 once, yielding:

$$\left\lfloor \frac{2021}{3} \right\rfloor + \left\lfloor \frac{2021}{5} \right\rfloor - \left\lfloor \frac{2021}{3 \times 5} \right\rfloor - \left\lfloor \frac{2021}{3 \times 7} \right\rfloor - \left\lfloor \frac{2021}{5 \times 7} \right\rfloor + \left\lfloor \frac{2021}{3 \times 5 \times 7} \right\rfloor = 809$$

6. Consider the center of AB as M, and the center of one circle as O. O will be on the same line with M. Using the Pythagorean Theorem on triangle AOM to find $\overline{OM}$, with $\overline{AO} = 1$ and $\overline{AM} = 1$:

$$\overline{OM}^2 + (\frac{1}{2})^2 = 1$$

$$\overline{OM}^2 = \frac{3}{4}$$

$$\overline{OM} = \frac{\sqrt{3}}{2}$$

Denote $\angle AOB$ as θ:

$$sin(\frac{\theta}{2}) = \overline{AM} = \frac{1}{4}$$

$$\theta = arcsin(\frac{1}{2}) \times 2$$

$$\theta = \frac{\pi}{3}$$

The area of intersection of two circles is twice the difference between the area of the sector AOB and triangle AOB.

The area of sector is:

$$\frac{\theta}{2} \times 1^2 = \pi/6$$

The area of triangle is:

$$\frac{\overline{AB} \times \overline{OM}}{2} = 1 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4}$$

The difference between the area of the sector AOB and triangle AOB becomes:

$$\frac{\pi}{6} - \frac{\sqrt{3}}{4}$$

Twice the difference yields:

$$\frac{\pi}{3} - \frac{\sqrt{3}}{2}$$