Sketching Graphs of Sinusoidal Functions

Summary

A sinusoidal function is a transformation of the basic curves y= cos(x) or y = sin(x). In Calculus, we use radian mode exclusively.

The domain of a basic sinusoid is all real numbers. The range is the interval $1 \leq y \leq 1$.

The period is 2π.

A tangent function is a transformation of y = tan(x). The key properties of this function follow from the identity tan(x) = sin(x)/cos(x).

Since cos(x) = 0 when $x = \frac{\pi}{2} + n\pi$ where n is an integer, and since sin(x) is not zero at these values, $x = \frac{\pi}{2} + n\pi$ are vertical asymptotes of the tangent graph. The range of the tangent function is all real numbers.

The period of tan(x) is π

The key properties of y = cot(x), y = sec(x), and y = csc(x) follow from the reciprocal identities cot(x) = 1/tan(x), csc(x) = 1/sin(x), and sec(x) = 1/cos(x).

You should know the exact values of sin(x) and cos(x) for the special angles (0, 30, 45, 60, and 90 degrees). You should be able to use properties of the trig functions to compute similar values for all six trig functions in all quadrants. The first quadrant values are listed below in a special way that should help make memorizing these values easy.

x =	0	π/6	π/4	π/3	π/2
sin(x)	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
cos(x)	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$

Problems

For each function in problems 1 – 4 sketch a graph without a calculator, find the domain and range, and identify the period.

1. $y = 3\sin(2\pi x) + 1$
2. $y = 4 - 2\cos(3x)$
3. $y = -\tan(x)$
4. $y = \frac{1}{4}\sec\left(\frac{\pi x}{2}\right)$

5. Find all solutions to $2\sin(x) = 3\sin(x)$.
6. Find all solutions between 0 and 2π to the equation $\sin(2x) = \cos(2x)$.
7. Find all solutions between 0 and 2π to the equation $\cos^2(x) = \cos(x)$. The notation $\cos^2(x)$ means $(\cos(x))^2$.
8. Answer this question without a calculator. How many intersections are there between the graph y = sin(x) and each of the following lines. It will be useful to know that the line y = x intersects y = sin(x) at just one point. Give reasons for your answers.
   a) y = 1/3
   b) y = 2x
   c) y = x - 2π
   d) y = 1.2
   e) x = 1/3
   f) y = x/2