Sketching Graphs of Sinusoidal Functions

Problems

For each function in problems 1 – 4 use your calculator to check your graph.

1. The domain is all reals, the range is $-2 \leq y \leq 4$, the period is 1.
2. The domain is all reals, the range is $2 \leq y \leq 6$, the period is $2\pi/3$.
3. The domain is all reals except for $x = \frac{\pi}{2} + n\pi$ for any integer n. The range is all reals, the period is π.
4. The domain is all reals except for $x = 1 + 4n$ for any integer value of *n*. The range is $|y| \geq \frac{1}{4}$ and the period is 4.

5. The equation is equivalent to sin(x) = 0, so the solutions are x = nπ for any integer n.
6. Rewrite the equation as $\frac{\sin(2x)}{\cos(2x)} = 1$ to see that the problem is equivalent to tan(2x) = 1. The solutions are values of x for which $2x = \frac{\pi}{4} + n\pi$ or $x = \frac{\pi}{8} + \frac{n\pi}{2}$. When n = 0, 1, 2, and 3 we get solutions in the desired domain: $x = \frac{\pi}{8}, \frac{5\pi}{8}, \frac{9\pi}{8}, \frac{13\pi}{8}$.
7. Rewrite the equation in factored form as $\cos(x)\left(\cos(x) - 1\right) = 0$. The solutions are $x = 0, \frac{\pi}{2}, \frac{3\pi}{2}, 2\pi$.
8. a) Since 1/3 is in the range of y = sin(x) there are infinitely many intersections with y = 1/3.
   b) Since y = x intersects the graph once so does the steeper line y = 2x.
   c) Since y = x - 2π is parallel to y = x it also intersects sin(x) just once at (2π,0).
   d) Since 1.2 is not in the range of y = sin(x) the horizontal line y = 1.2 does not intersect the graph at all.
   e) x = 1/3 intersects the graph just once since y = sin(x) is a function.
   f) Since y = x/2 obviously intersects the sine wave at the origin. The point (π, π/2) lies on the line and since π/2 > 1 the line must intersect the sine graph somewhere between x = 0 and x = π. There is a corresponding intersection in the fourth quadrant since sin(x) is symmetrical through the origin. Therefore, this line intersects y = sin(x) exactly three times.