Sketching Graphs of Exponential Functions

Answer

You can check all your graphs and intercepts with your calculator.

1. $y = 4e^{3x} - 8$
    a. Sketch a graph.
    b. The domain is all real numbers, the range is *y* > -8 since the graph has been shifted down 8 units.
    c. The horizontal asymptote is y = -8.
    d. The *y*-intercept is (0, -8). The x-intercept is $\left(\frac{\ln(2)}{3}, 0\right)$
2. $y = 2 - 3^{-x}$
    a. Sketch a graph.
    b. The domain is all real numbers; the range is y < 2.
    c. The horizontal asymptote is The horizontal asymptote is *y* = 2.
    d. The y-intercept is (0, 1). The x-intercept is $-\log_3(2), 0$ or $\left(-\frac{\ln(2)}{\ln(3)}, 0\right)$
3. Rewrite the equation as $e^x = xe^x e^2$ so that $e^x \;\; 1 - xe^2 \;\; = 0$. The only *x*-intercept is $x = 1/e^2$
4. Solve $0 = 2x^2 e^x - x^3 e^{x-1}$ by factoring: $0 = x^2 e^x \;\; 2 - xe^{-1}$. There are two *x*-intercepts: *x* = 0 and *x* = 2*e*. To decide whether or not the graph has a high point between these intercepts consider the local behavior of the function near these intercepts.

    When x is a small positive number $y = x^2 e^x \;\; 2 - xe^{-1}$ is positive. When x is a little smaller than 2*e* the *y*-values are also positive. But if the graph is positive between these intercepts it must increase to some maximum value and then decrease to zero.
5. The exponential function decreases everywhere while the cubic function increases. The two curves are separated at x = 0 and so must cross exactly once somewhere between x = 0 and x = 1.
6. The exponential function $y = e^{-x}$ is positive and decreasing. The function $y = -e^{-x}$ has been reflected across the *x*-axis and so is negative and increasing. Therefore, the function $y = 1 - e^{-x}$ must intersect $y = e^{-x}$ exactly once.