

Solving Inequalities

1. $y = 4x^5 - 6x^4 - 3x^2 + 3 > x^3 + 2x + 2$ when $-0.628 < x < 0.314$ and $x > 1.895$
2. $\ln(x) > \sin(x)$ when $x > 2.219$
3. $\frac{x^3 - 2x}{x^3 - 3x} > 0$ on the intervals $(-\infty, -1.732) \cup (-1.414, 1.414) \cup (1.732, \infty)$.
4. To solve $x^2 + 5x - 1 < 3x^2 + 2x$ rewrite the inequality as $0 < 2x^2 - 3x + 1 = (2x - 1)(x - 1)$.
The quadratic is negative between its x-intercepts so the solution is $x < 1/2$ and $x > 1$.
5. To solve $x^3 - 4x^2 + x + 6 \geq 0$ use the Factor Theorem to factor the cubic into $(x + 1)(x - 3)(x - 2) \geq 0$. The solution are the intervals $[1, 2] \cup [3, \infty)$.
6. To solve the inequality $\frac{x^3 - 2x}{x^3 - 3x} > 0$ factor into $\frac{x(x^2 - 2)}{x(x^2 - 3)} > 0$. The common factor of x in the numerator and denominator corresponds to a hole on the graph and does not affect the intervals where this rational function is positive and negative. The graph of this fraction has vertical asymptotes at $x = \pm\sqrt{3}$ and zeros at $x = \pm\sqrt{2}$. Since the long-range behavior is positive, the fraction is positive on the intervals $(-\infty, -\sqrt{3}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{3}, \infty)$.