

## Sketching Graphs Using Transformations

### Summary

Adding, Subtracting, Multiplying and Dividing a function by a number has the effect of transforming the graph of the function in various ways.

Suppose  $y = f(x)$  is a function and  $a > 0$  is a positive constant.

$y = f(x) + a$       Shifts the graph  $a$  units upward  
 $y = f(x) - a$       Shifts the graph  $a$  units downward  
 $y = f(x + a)$       Shifts the graph  $a$  units to the left  
 $y = f(x - a)$       Shifts the graph  $a$  units to the right

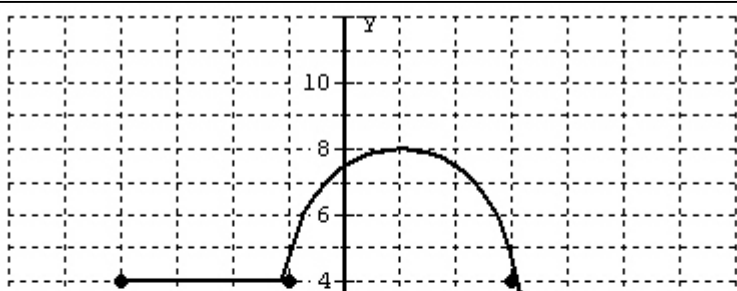
$y = af(x)$       If  $a > 1$  this stretches the graph vertically.  
                     If  $a < 1$  this compresses the graph vertically.  
 $y = f(ax)$       If  $a > 1$  this compresses the graph horizontally.  
                     If  $a < 1$  this stretches the graph horizontally

$y = -f(x)$       Reflects the graph across the  $x$ -axis  
 $y = f(-x)$       Reflects the graph across the  $y$ -axis

### Problems

1. What sequence of transformations will change  $y = |x|$  into  $y = 3|1 - x|$
2. What sequence of transformations will change  $y = 1/x$  into  $y = 4 + 2/(x + 16)$
3. The graph of  $y = 2^x$  is shifted 4 units down, then stretched vertically by a factor of 3, and then reflected across the  $y$ -axis. What is the equation of the new function?
4. The graph of  $y = \ln(x) + |x|$  is shifted 1 unit to the right, then reflected across the  $x$ -axis, then compressed vertically by a factor of 2, and then shifted vertically downward 5 units. What is the equation of the new function?

The following questions refer to the graph of the function  $y = f(x)$  defined on  $-8 \leq x \leq 12$  shown here. In each problem, sketch the graph of the given transformation of  $f$ .



5.  $y = 2f(2x) + 4$

6.  $y = 8 - f(x/4)$

7.  $y = f(2 - x)$

8.  $y = 6 - 2f(x)$