Simplifying Algebraic Expressions Using Properties of Exponents

Properties of Exponents	
The Multiplication Property	$a^n \times a^m = a^{n+m}$
The Division Property	$\dfrac{a^n}{a^m} = a^{n-m}$
The Power Property	$\left(a^n\right)^m = a^{nm}$
The Distributive Property	$(ab)^n = a^n b^n$
The Zero Power Property	$a^0 = 1$ for any quantity $a \neq 0$
The Negative Power Property	$a^{-n} = \dfrac{1}{a^n}$
The Fractional Power Property	$a^{n/m} = \sqrt[m]{a^n}$ if $a > 0$

Example: Write as a simplified radical" $\sqrt{\sqrt{8}}$

solution: $\sqrt{\sqrt{8}} = \left(\left(8\right)^{1/2}\right)^{1/2} = 8^{1/4}$. Therefore, as a simplified radical, this expression is $\sqrt[4]{8}$

Problems

1.   Without a calculator, compute the exact value of each of the following.

   a. 9^{-1} b. $25^{3/2}$ c. $27^{2/3}$

   d. $16^{-1/2}$ e. $\dfrac{18^{3/2}}{18^{1/2}}$ f. $64^{1/2} - 64^{1/6} - 64^{1/3}$

   g. $\left(\dfrac{9}{4}\right)^{-1/2}$ h. $(20)^{1/3}(20)^{1/2}(20)^{1/6}$ i. $(-8)^{4/3}(32)^{-2/5}$

2.   Simplify each of the following expressions as much as possible. Assume all variables are non-negative.

   a. $\left(\dfrac{a^{3/2}}{a^{2/3}}\right)^2$ b. $\left(\dfrac{a^2}{b^3}\right)\left(\dfrac{b^5}{a^3}\right)$ c. $\left(x^{-3}y^3z^6\right)^{2/3}$

   d. $\left(16p^{4/3}q^4r^8\right)^{3/4}$ e. $\dfrac{r^{3/2}s^{2/3}}{r^{1/2}s^{1/3}} \times \dfrac{r^{5/2}s^{-5/3}}{r^{7/2}s^{5/3}}$ f. $\left(xy^5\right)^{1/2}\left(x^2y^6\right)^{1/4}$

3. Simplify each of the following as much as possible. Do not leave negative exponents in your answer. Assume all variables are non-negative.

    a. $\left(\dfrac{12^{3/4}}{12^{-1/2}}\right)^{-1/2}$

    b. $\left(25^{3/4}x^{1/2}y^2\right)^2$

    c. $\left(12^{3/2}\right)\left(3^{3/2}\right)$

    d. $\left(\dfrac{a^4b^2}{c^6}\right)^{-1/2}$

    e. $\dfrac{18^{3/2}}{2^{3/2}}$

    f. $\left(\dfrac{3^{3/2}a^3b^{-2}}{3ab^{-1}}\right)^{-2}$

4. Rewrite each of the following using fractional exponents instead of radicals and then simplify as much as possible. Assume all variables are non-negative.

    a. $\sqrt{x^3}$

    b. $\sqrt[3]{y^2z}$

    c. $\sqrt{x\sqrt[3]{y}}$

    d. $\sqrt[3]{x^4}\,\sqrt[3]{x^2}$

    e. $\dfrac{\sqrt[4]{x^3}}{\sqrt[2]{x}}$

    f. $\sqrt[3]{\sqrt[3]{x^7}}$